

CALCULUS I, TEST II

MA 126

Closed Book, No Calculators

100 points

Name \_\_\_\_\_

Fall, 2008

**Part 1.** Part 1 consists of 6 questions. Do your work and clearly write your answer in the space provided after each question. Little or no partial credit is awarded for this part of the test. (5 points each)

1. Find the equation of the plane that passes through  $P(-1, 3, -2)$  and is parallel to the plane  $2x - y + 4z = 7$ .

Answer: \_\_\_\_\_

2. Let  $\vec{a} = \langle 1, 1, -1 \rangle$  and  $\vec{b} = \langle 3, -2, 1 \rangle$ . Find

(a)  $\text{comp}_{\vec{a}} \vec{b}$

Answer: \_\_\_\_\_

(b)  $\overrightarrow{\text{proj}}_{\vec{a}} \vec{b}$

Answer: \_\_\_\_\_

- 3 Find the angle between the vectors  $\vec{u} = \vec{i} + 2\vec{j} - 3\vec{k}$  and  $\vec{v} = \vec{i} + \vec{j} + \vec{k}$ . You may leave your answer in the form  $\theta = \cos^{-1}(y)$ .

Answer: \_\_\_\_\_

4. A constant force  $\vec{F} = 3\vec{i} + 5\vec{j} + 10\vec{k}$  moves an object along the line segment from the point  $P(1,0,1)$  to the point  $Q(5,3,8)$ . Find the work done (include units) if the distance is measured in meters and the force in newtons.

Answer: \_\_\_\_\_

5.  $16x^2 + 16y^2 + 16z^2 - 128x + 32y = 5$  is the equation of a sphere. Find the radius.

Answer: \_\_\_\_\_

6. Find the distance between the point  $P(1,2,3)$  and the line  $L$  given by  $x = 2 + t$ ,  $y = 3 - t$ ,  $z = 5t$ .

Answer: \_\_\_\_\_

**Part 2. Part 2 consists of 5 problems worth 14 points apiece. Show all your work for full credit! Displaying only the final answer (even if correct) without the relevant steps is not enough.**

**Problem 1**

This problem has two separate questions (a) and (b). Answer each question.

- (a) Suppose two lines are given by the parametric equations given below. Determine whether these lines intersect. If they do intersect, find the point of intersection.

$$L_1: x = -1 + 2t, y = 3t, z = 4 - 3t$$

$$L_2: x = 3 + s, y = 1 - s, z = 1$$

- (b) Is the line given by  $\vec{r} = \langle 1, 2, 3 \rangle + t \langle -2, 2, -4 \rangle$  parallel to the line given by  $\vec{r} = \langle -1, 3, -4 \rangle + t \langle 1, -1, 8 \rangle$ ? Why or why not?

**Problem 2**

Let  $P(1, 4, 6)$ ,  $Q(-2, 5, -1)$ , and  $R(1, -1, 1)$  be three points in  $R^3$ .

- (a) Find the equation of the plane through the points  $P$ ,  $Q$ , and  $R$ .
- b. Find the area of  $\Delta PQR$ , the triangle with vertices at  $P$ ,  $Q$ , and  $R$ .

3. (a) Find the parametric equations and symmetric equations for the line passing through the points  $P(10,3,1)$  and  $Q(5,6,-3)$ . Label which is which.

- (b) At what point does this line intersect the  $xy$ -plane?

**Problem 4**

Let  $x + y - z = 2$  and  $2x - y + 3z = 1$  be the equations of two planes.

a. Find one point that lies on the line of intersection of the two planes.

b. Find the vector equation of the line of intersection of the two planes.

**Problem 5**

A particle is traveling along the space curve

$$\vec{r} = \left\langle \arctan t, e^{-t}, \frac{\ln t}{t} \right\rangle$$

when the time  $t > 0$ .

- a. Evaluate  $\lim_{t \rightarrow \infty} \vec{r}(t)$  to see what point the particle approaches after a long time.

- b. Determine a unit tangent vector to the curve at the point where  $t = 1$ .